MACROSCALE SURFACE ROUGHNESS AND FRICTIONAL RESISTANCE IN OVERLAND FLOW

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ABSTRACT

The hydraulics of overland flow on rough granular surfaces can be modelled and evaluated using the inundation ratio rather than the flow Reynolds number, as the primary dimensionless group determining the flow behaviour. The inundation ratio describes the average degree of submergence of the surface roughness and is used to distinguish three flow regimes representing partially inundated, marginally inundated and well-inundated surfaces. A heuristic physical model for the flow hydraulics in each regime demonstrates that the three states of flow are characterized by very different functional dependencies of frictional resistance on the scaled depth of flow. At partial inundation, flow resistance is associated with the drag force derived from individual roughness elements and therefore increases with depth and percentage cover. At marginal inundation, the size of the roughness elements relative to the depth of flow controls the degree of vertical mixing in the flow so that frictional resistance tends to decrease very rapidly with increasing depth of flow. Well-inundated flows are described using rough turbulent flow hydraulics previously developed for open channel flows. These flows exhibit a much more gradual decrease in frictional resistance with increasing depth than that observed during marginal inundation.

A data set compiled from previously published studies of overland flow hydraulics is used to assess the functional dependence of frictional resistance on inundation ratio over a wide range of flow conditions. The data confirm the non-monotonic dependence predicted by the model and support the differentiation of three flow regimes based on the inundation ratio. Although the percentage cover and the surface slope may be of importance in addition to the inundation ratio in the partially and marginally inundated regimes, the Reynolds number appears to be of significance only in describing well-inundated flows at low to moderate Reynolds numbers. As these latter conditions are quite rare in natural environments, the inundation ratio rather than the Reynolds number should be used as the primary dimensionless group when evaluating the hydraulics of overland flow on rough surfaces.

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INTRODUCTION

Although several well-known field studies have considered overland flow hydraulics on rough surfaces (i.e. Emmett, 1960; Dunne and Dietrich, 1980; Abrahams *et al*., 1986) and more recent work has focused on the relative importance of various contributions to frictional resistance (i.e. Rauws, 1988; Parsons *et al*., 1990; Abrahams and Parsons, 1991, Gilley *et al*., 1991, 1992; Weltz *et al*., 1992; Abrahams *et al*., 1994; Gilley and Kottwitz, 1995; Katz *et al*., 1995), a systematic theory for evaluating these surface flow regimes has not yet been developed. Previous empirical work has thus largely relied on analogy with Darcy–Weisbach pipe flow experiments to estimate the functional relationship between flow characteristics and frictional resistance. In applying this methodology, overland flow hydraulics are assessed by considering the Darcy–Weisbach friction factor as a function of Reynolds number. Early efforts in this regard, as exemplified by the field experimental and laboratory work of Emmett (1970), sought to identify a laminar resistance regime, analogous to that which is observed for low to moderate Reynolds number flow through pipes, in which the friction factor can be directly predicted from the Reynolds number of the flow. Similarly, Dunne and Dietrich (1980) based the analysis of their field data on the assumption of an explicit dependence of friction factor on Reynolds number and suggested that the variability in the coefficient associated with this dependence is due to differences in the surface roughness at each site. In principle, this approach should be reliable for a range of flow conditions which

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are hydrodynamically similar to those associated with one-dimensional flow through pipes. It is an especially appropriate model when the surface roughness is small relative to the depth of flow or, in the case of large scale roughness, when the Reynolds number is very low so that inertia is negligible. In practice, however, those conditions are rarely found in overland flow over natural surfaces. More often, the disturbed (though not necessarily turbulent) character of the flow and the variable influence of roughness as flow depths increase both contribute to a significantly more complex flow system than the one-dimensional pipe flow conditions for which the Darcy–Weisbach theory was derived.

As recognized in more recent field and laboratory studies, e.g. Abrahams *et al*., 1986; Rauws, 1988; Gilley *et al*., 1992; Gilley and Kottwitz, 1995), overland flow in the presence of macroscale roughness may exhibit a non-monotonic variation in frictional resistance with increasing depth, which is not at all analogous to the functional dependence of the friction factor on the Reynolds number associated with flow through pipes. Furthermore, the systematic character of this trend may only become fully apparent when the frictional resistance is considered as a function of the inundation ratio (i.e. the ratio of the mean flow depth to the average roughness height), rather than the flow Reynolds number, due to the disturbed, 'quasi-turbulent' character of the flow. Despite this, virtually all field and experimental studies continue to report frictional resistance as a function of Reynolds number rather than a roughness or inundation ratio when presenting data describing overland flow hydraulics. The purpose of this paper is therefore to pose a broad model for the hydraulics of overland flow by assessing the dependence of the friction factor on an inundation ratio at three stages of progressive flow inundation. The development of the model is prefaced by a brief review of the physical variables which may contribute to flow resistance and some of the possible dimensionless groupings arising from those variables. Following the presentation and discussion of the model, previously published field and laboratory data are re-evaluated in terms of the frictional resistance as a function of an estimated inundation ratio for each flow, and the results are compared with the trends predicted by the model. In contrast to previous work, the trends in the frictional resistance associated with very different flow systems can be directly compared once the flow depth is scaled by the roughness of the surface. The comparison demonstrates both the ultility and limitations of the proposed heuristic model, thereby providing direction for future theoretical and experimental work.

PHYSICAL VARIABLES AND DIMENSIONLESS GROUPINGS

Several physical factors may contribute to and can accordingly be used as a basis for evaluating the hydraulics of overland flow on natural surfaces. In general, the flow is primarily gravity-driven in the downstream direction and the internal resistance to flow (in contrast to the 'external' resistance imposed by the rough boundary) is derived from the viscosity of the fluid. The dominant physical variables associated with a one-dimensional, gravity-driven free surface flow are given by: (1) the downslope component of the gravitational acceleration, $g \sin \theta$, where θ is the angle of the surface slope; (2) the fluid density and dynamic viscosity, ρ and μ, respectively (which in some cases may depend upon the presence of entrained solids); (3) the depth of flow, d; and (4) the average flow velocity V. These variables arise directly from the one-dimensional solution of the momentum balance equations for a non-accelerating flow down an incline, and although this solution is for a greatly simplified flow system it nonetheless characterizes a fundamental physical balance. When the volume of fluid is locally conserved, an alternative variable which is also often considered is the specific discharge $q = Vd$. Although in most hydraulic analyses V and d represent primary variables, in many field and laboratory studies of overland flow it is often q which is the measured variable and V is calculated from the average depth of the flow (e.g. Emmett, 1970; Abrahams *et al*., 1986), which may itself be derived from the weight of the fluid (e.g. Savat, 1980).

The roughness of the ground surface also strongly influences the hydraulics of the flow field, particularly when the surface roughness elements are not completely or are only marginally inundated. However, as most natural surfaces consist of a range of particle sizes exposed on the surface, uniquely specifying the surface roughness for the purposes of general modelling is clearly problematic. One would, though, anticipate that either the mean or median particle size of the surface or some other primary measure of the particle size distribution, e.g. D_{85}, is appropriate for identifying the principal length-scale representing the surface roughness. This length-scale can then be used to scale the mean depth of flow and thereby provide a first-order

indication of the degree of surface inundation. In the physical modelling presented below, (1/2) $D_{50} = k$ will be used to specify the primary length-scale of the surface roughness as a very general criterion for distinguishing flow regimes. The choice of this particular measure is based in part on practicality, as this is the surface roughness parameter which is most commonly reported in field and laboratory studies and thus represents a relatively consistent measure for scaling data from very different flow systems. It is nevertheless recognized that measures of additional factors contributing to flow resistance (e.g. percentage gravel or vegetation cover) and/or more sophisticated descriptions of the particle size distribution (e.g. mean diameter weighted by stone surface area and perimeter length as suggested by Dunkerley (1995)) may in many cases be necessary in order to fully specify the problem.

Following Buckingham's π theorem (see, for example, Langhaar (1951) for a more detailed discussion of this theorem), the seven physical quantities g, ρ, μ, d, V, $\sin\theta$ and k can be replaced with four characteristic dimensionless groups, by eliminating the three basic dimensional measures mass (M), length (L) and time (T). The choice of dimensionless groups is not unique and so should be guided by physical insight to ensure an appropriate scheme for modelling.

One can anticipate that both gravitational and inertial effects may contribute to the flow, so that a natural dimensionless grouping is given by the ratio of these two factors, i.e.

$$\frac{\text{gravitational stress}}{\text{inertial stress}} = \frac{\rho g d \sin\theta}{\rho V^2} \tag{1}$$

This is actually the friction factor, which for free surface flows is generally posed as:

$$f = \frac{8 g d \sin\theta}{V^2} \tag{2}$$

The factor of eight is historical and reflects the extension of the Darcy–Weisbach pipe flow theory, which is based on a hydraulic diameter, to the case of free surface flows which are evaluated in terms of a hydraulic radius. It is thus somewhat arbitrary, but is included here to be consistent with published field and laboratory data.

The Reynolds number, which measures the relative significance of inertia and viscosity in the flow field, can also be constructed from the variables and is given by

$$Re = \frac{\text{inertial stress}}{\text{viscous stress}} = 4\frac{\rho V^2}{\mu V/d} = 4\frac{\rho d V}{\mu} \tag{3}$$

where the factor of four is also of historic origin. This dimensionless group is highly significant in the assessment of frictional resistance in pipe flow, as illustrated in the classic Moody diagram (see Figure 1). In the 'laminar' regime, the friction factor is inversely proportional to the Reynolds number, and at the point where the flow becomes unstable to turbulence (i.e. at Reynolds numbers of approximately 2000) this relationship breaks down. At higher Reynolds numbers, the dimensionless group which determines the frictional resistance is the roughness ratio, which is given by $\varepsilon = e/d$, where e is the characteristic scale for the roughness of the pipe walls. This dimensionless grouping, which has long been recognized as significant in pipe flow and river hydraulics, has been virtually neglected in quantitative field studies of overland flow. In the analysis which follows, the inverse of this ratio will be used, as given by:

$$\Lambda = \frac{d}{k} \tag{4}$$

where k represents the characteristic roughness scale for the surface. This ratio thus represents the average degree of inundation of the rough surface in a dimensionless form. One can therefore anticipate that this should be a significant variable for a wide range of flows, excepting of course the case of a well-inundated surface at

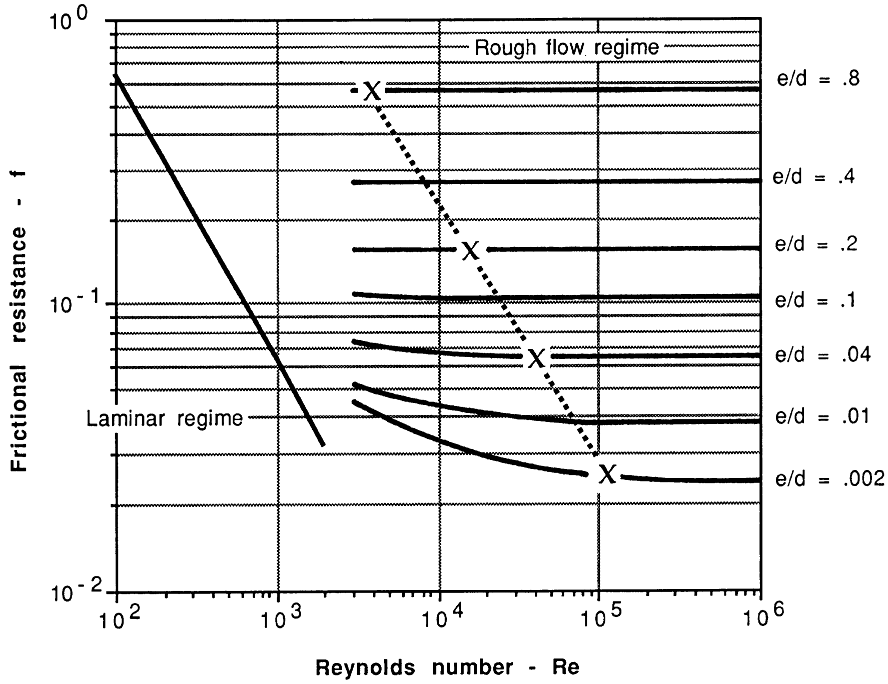

Figure 1. Frictional resistance as a function of Reynolds number for one-dimensional flow through pipes in the laminar and rough flow turbulent regimes. In the laminar regime, the frictional resistance is inversely proportional to the Reynolds number with $f = 96/(Re)$, while in the turbulent regime it is determined by the roughness ratio (see, for examples, Shames (1993) for derivations of these relationships). the dashed line suggests an apparent 'laminar' flow behaviour obtained by varying the roughness ratio, which for overland flow would correspond to varying the depth of flow over a given surface

low to moderate Reynolds numbers, which is neither disturbed by local boundary roughness nor unstable to turbulence.

Although the three dimensionless groups which have been identified thus far, i.e. the friction factor f, the Reynolds number Re, and the inundation ratio Λ, are analogous to the dimensionless groupings used for evaluating pipe flow hydraulics, there are nevertheless two major discrepancies between the empirical studies upon which the Darcy–Weisbach theory for pipe flow is based and field studies of overland flow.

Firstly, the pipe flow experiments consider roughness ratios $\varepsilon \ll 1$, whereas in overland flow on rough surfaces $\varepsilon \geq 1$. Thus, in a straight pipe at moderate Reynolds numbers, the flow lines always remain approximately parallel to the boundary, so that the viscous stress dominates the flow and the frictional resistance varies as the inverse of the Reynolds number within this regime. When the roughness elements begin to protrude substantially into the flow field so that the roughness ratio approaches one, we would expect the flow lines to be diverted away from simple parallel flow. In this latter case, we expect the frictional resistance in the flow to be derived from both the viscous stress and the inertial stress when the Reynolds number is of order one, and primarily from the inertial stress as the Reynolds number increases. Therefore, the Reynolds number at which the roughness ratio becomes the dominant dimensionless group should be much smaller for overland flow in the presence of macroscale roughness than it is for classical pipe flow, which is generally associated with a roughness ratio ε of 0·04 or less. In other words, the inherently three-dimensional character of very thin flow over a rough surface effectively introduces inertial stress as a dominant source of resistance to the downstream flux at a much lower Reynolds number than that which is generally associated with a transition to turbulence in an otherwise uniform flow.

The second shortcoming associated with the standard use of Darcy–Weisbach theory to evaluate overland flow hydraulics is a consequence of experimental design. In laboratory studies of pipe flow, the roughness ratio

can be kept constant for a given set of experiments as both the roughness length and the pipe diameter are constants for a particular experimental set-up. Variations in the flow rate are obtained by changing the pressure drop across the length of the pipe. In field experiments of overland flow this is not possibe, so that both the Reynolds number and the roughness ratio vary with each downslope cross-section and with each run. One could only set up a field experiment similar to the pipe flow studies by varying the ground slope of a plot for each subsequent run and simultaneously adjusting the discharge to achieve a constant inundation ratio. The significance of this discrepancy in experimental technique is that when the results which are obtained by increasing the discharge are evaluated in terms of the frictional resistance as a function of the Reynolds number, one may obtain a spurious correlation which actually arises from the decrease in the roughness ratio as the flow depth increases with Reynolds number (as is illustrated in Figure 1), rather than a true dependence of frictional resistance on Reynolds number. The dashed sloping line in Figure 1 indicates the apparent dependence of the friction factor on the Reynolds number which might be found due to changes in the flow depth, which inadvertently also alter the roughness ratio, during sequential experimental runs.

An additional dimensionless group which could be considered in this analysis is the Froude number, i.e.

$$Fr = \left(\frac{V^2}{gd}\right)^{1/2} \tag{5}$$

which characterizes the flow speed relative to the speed of surface waves and may be of importance whenever a flow possesses a free surface. If wave drag makes a significant contribution to flow resistance, as argued by Abrahams and Parsons (1994), then one would expect the Froude number to highlight this dependence. In considering this, though, one must recognize that in states of partial inundation the local Froude number of the flow will most likely be highly variable, with numerous transitions between supercritical and subcritical flow, as the flow is diverted around surface roughness elements (e.g. Bunte and Poesen, 1994). These transitions will conserve flow momentum, but dissipate energy, thus increasing the effective frictional resistance to flow, so that the tendency for wave drag will be strongly related to the inundation ratio Λ and other measures of the surface roughness, such as the percentage stone cover. It should also be noted that:

$$Fr = \frac{8\sin\theta}{f} \tag{6}$$

so that if the friction factor f is posed as the dependent dimensionless quantity, then one need not consider the Froude number explicitly, but can rather simply choose sin θ as the independent dimensionless grouping. The interdependence of the Froude number and the friction factor also emphasizes that, for a surface with a given slope angle, the Froude number is actually determined by the hydraulics of the flow field rather than being an independent variable which uniquely contributes to setting the flow regime.

The following dimensionless groupings are therefore all relevant in the assessment of overland flow hydraulics:

$$f = f(Re, \Lambda, \sin\theta) \tag{7}$$

and employ all of the physical variables identified in the first part of this section without redundancy. Although it is recognized that higher-order descriptors of the surface roughness may be important in fully determining the flow hydraulics, the simplest set of parameters will be used here to correlate experimental and field data with the friction factor. In many cases, the dependence on the Reynolds number will be very small because the flow is either partially or slightly inundated or is fully disturbed by the surface roughness, so that the flow regime is similar to turbulent pipe flow in which roughness dominates. Furthermore, one would expect that there may be very little explicit dependence on sin θ independent of the roughness ratio, particularly when the slope is reasonably small. If both of these approximations are valid, then for a given surface geometry $f = f(\Lambda)$. The physical modelling presented below will be based on this assumption of univariate dependence. Recognizing

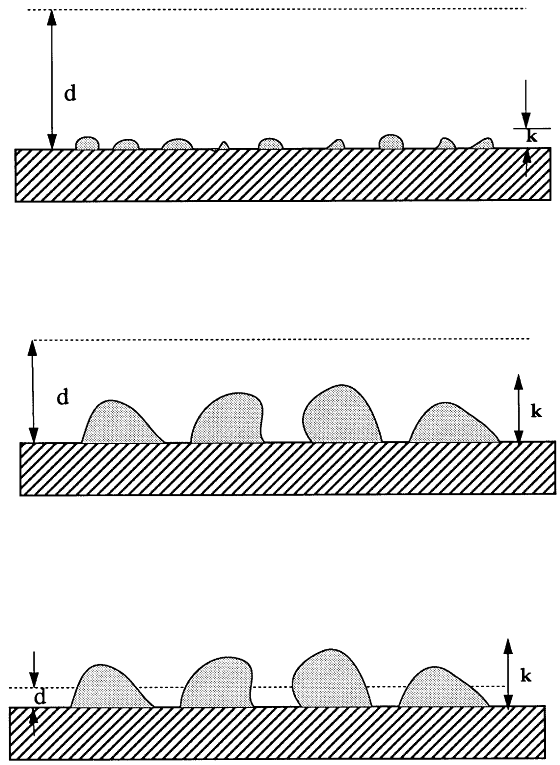

Figure 2. Well-inundated (top), marginally inundated (middle) and partially inundated (bottom) flow regimes

the limitations of this approach, however, the effects and apparent contributions of the neglected variables will be considered using multivariate regression analyses.

MODELLING OF SURFACE ROUGHNESS AND FRICTIONAL RESISTANCE

For the purposes of evaluating overland flow on a rough surface, three distinct flow regimes can be identified based on the degree of inundation of the roughness elements, as illustrated in Figure 2. In the first regime, the elements are very small relative to the flow depth (i.e. $k \ll d$) so that they introduce a boundary scale roughness, but do not necessarily disturb the entire velocity profile. This regime is generally analogous to the ‘rough flow’ regime in open channel flow hydraulics, in which the roughness elements protrude through the viscous sublayer at the boundary and thus introduce resistance into the flow but do not significantly alter the one-dimensional character of the flow field. The second regime is one in which the elements are inundated but the disturbance associated with their presence affects the entire vertical flow field, due to the relatively shallow flow depth, rather than being localized in the vicinity of the boundary. The depth of the water is greater than the characteristic height of the roughness elements, so that $k < d$; however, d/k is of order one, imparting a multidimensional character to the velocity field. The third regime represents the case in which the flow is quite shallow, so that the roughness elements protrude through the flow. The depth of the water is less than or equal to the height of the roughness elements, with $d < k$. Changes in the depth of flow are not only associated with simple variations in resistance to flow, but also contribute to changes in the hydraulic radius, which in this regime is generally not simply equal to the depth of flow (Abrahams and Parsons, 1990).

The first of these regimes actually represents flow in the presence of microscale rather than macroscale roughness, so that one can rely on standard hydraulic analyses for estimating the approximate relationship

between flow resistance and surface roughness in these flows. One is therefore interested in distinguishing both laminar and turbulent flows and, in the turbulent range, between hydraulically smooth versus transitional versus hydraulically rough flow. For the case of strictly laminar flow, which in this instance is characterized both by a low to moderate Reynolds number and by $d \ll k$, there should be no relationship between the roughness and the frictional resistance as this is the regime which is truly analogous to the 'laminar' regime of one-dimensional pipe flow. However, this regime is actually very rare in overland flow as it requires a very deep flow so that all of the surface roughness elements are well-inundated, but with a sufficiently slow velocity to ensure laminar flow. More commonly, the well-inundated flows represent hydraulically rough flows within the turbulent range, such that the relationship between surface roughness and frictional resistance can be determined using techniques developed for open channel hydraulics.

In particular, in a turbulent flow, the shear stress in the fluid represents a balance between the weight of the fluid and the resistance to flow, as given by:

$$\tau = \rho g \sin\theta (d - y) = \rho \nu_{\mathrm{T}} \left(\frac{\mathrm{d}u}{\mathrm{d}y} \right) \tag{8}$$

where u is the downstream component of the flow velocity, y is the vertical coordinate and ν_{T} is the turbulent kinematic viscosity or 'eddy viscosity' of the flow field. Prandtl's mixing length hypothesis states that:

$$\nu_T = l^2 \frac{\mathrm{d}u}{\mathrm{d}y}$$

so that (9)

$$l \frac{\mathrm{d}u}{\mathrm{d}y} = \sqrt{gd \sin\theta} \left[1 - \frac{y}{d} \right]^{1/2}$$

where l is the mixing length, which near the boundary can be approximated as $l \sim \kappa y$, with $\kappa \sim 0{\cdot}44$. Equation 9 can then be integrated in the vicinity of the boundary to obtain the standard turbulent velocity profile:

$$u \sim V^* \left[\frac{1}{\kappa} \ln(y^*) + B \right] \tag{10}$$

where $V^* = \sqrt{\tau_0/\rho} = \sqrt{gd\sin\theta}$ is the shear velocity, $y^* = y/e = y/(2k)$, and B is a constant factor which is approximately equal to 8·5 in fully rough flow (see, for example, Shames (1993, pp. 716–719) for more discussion and detail). In evaluating flow hydraulics, one is primarily interested in the mean flow velocity and this can be derived from Equation 10 by integrating through the depth to obtain the specific discharge and dividing the result by the flow thickness, which gives:

$$V = V^* \left[\frac{1}{\kappa} \left(1\mathrm{n}\left(\tfrac{1}{2} \Lambda \right) - 1 \right) + B \right] \tag{11}$$

By substituting the definition of the friction factor via the relationship $V/V^* = \sqrt{8/f}$,one can write an expression which relates the frictional resistance directly to the surface roughness, as given by:

$$f^{-1/2} \sim 1{\cdot}64 + (0{\cdot}803) \ln(\Lambda) \tag{12}$$

This relationship was developed explicitly for rough turbulent flow and the majority of well-inundated flows which will be examined in the data analysis presented below fall into this range. Those which do not are transitional between smooth and rough turbulent flow and should be expected to exhibit some functional dependence of frictional resistance on Reynolds number, in addition to the dependence on roughness or inundation ratio given by Equation 12.

The second regime represents a state of flow in which the flow field is no longer one-dimensional; however, it is convenient to continue to model it using ideas of one-dimensional turbulent flow in which the vertical component of velocity associated with the diversion of flow over roughness elements effectively retards the downstream flow and thus contributes to flow resistance. In this regime, flow separation will also occur in the lee of the roughness elements, introducing additional resistance to flow. Rather than proposing a detailed and rigorous hydraulic theory for this regime, which would be well beyond the scope of the present work, one can suggest that the disturbance introduced into the flow field should scale roughly with the size of the roughness elements and effectively mix the flow over a similar length-scale. If this is the case, then a mixing length model, similar to that posed for turbulent flow in the vicinity of a boundary, as presented above, can be used to assess the relationship between surface roughness and frictional resistance. In other words, if the mixing length l scales directly with the surface roughness k, then $l \sim \kappa k$, and one can write:

$$\tau \sim \rho\kappa^2 k^2 \left(\frac{\delta u}{\delta y}\right)^2 = \rho g\,(d-y)\sin\theta \tag{13}$$

This can be solved directly for the velocity distribution $u\,(y)$ and from this an expression for the mean velocity V can be derived in the form:

$$V = \frac{2d}{5\kappa k}\,(gd\sin\theta)^{1/2} \tag{14}$$

so that the frictional resistance in terms of the inundation ratio is given by:

$$f = \frac{8gd\sin\theta}{V^2} = 50\left(\frac{k}{d}\right)^2 \kappa^2 \approx 10\Lambda^{-2} \tag{15}$$

This result is valid for the case where the total flow disturbance and the resistance introduced by that disturbance scale directly with the surface roughness. It therefore applies to flow configurations in which the flow depth is similar to the height of the roughness elements but is nonetheless sufficient to fully inundate the surface such that a well-developed layer of flow over the roughness can be identified.

As the flow becomes increasingly shallow, the effects of individual roughness elements on the flow resistance will become apparent, and this trend should continue as the surface is exposed and subject to only partial inundation. Therefore, an approximate relationship between the frictional resistance and the surface roughness can be derived by considering the drag introduced by the individual elements, as previously suggested by Phelps (1975) for the case of laminar flow over rough surfaces. In general, the drag force, F_D, associated with a particular element is given by:

$$F_D \sim C_D\,(\tfrac{1}{2})\,\rho V^2 A_F \tag{16}$$

where C_D is the coefficient of drag and A_F is the projected frontal area exposed to the flow field. The drag force can be directly related to the effective boundary shear stress by $\tau = nD$, where n is the number of elements per unit area. For a hemispherical particle of radius $r = k$, when the element is fully submerged $A_F = \frac{1}{2}\pi k^2$, and at partial submergence $A_F \approx 2kd$. In the case of a discretely bimodal distribution of particle sizes (as is true of many laboratory studies in which the percentage cover of the larger size fraction is varied), the number of particles per

unit area can be written as: $\underline{P}/(\pi r^2) = \underline{P}/(\pi k^2)$, where $\underline{P}$ is the fractional cover or portion of the surface covered by the larger particles.

Based on the foregoing arguments, the shear stress at the boundary is approximately given by:

$$\tau = nC_D \rho V^2 \mathrm{MIN}\left[\frac{\pi}{4}k^2, kd\right] \tag{17}$$

where the notation MIN[a,b] refers to the minimum possible value taken on by either variable a or b, say. The frictional resistance in the flow introduced at the boundary by the roughness can be approximated as:

$$f \sim \frac{8\tau}{\rho V^2} = 8nk^2 C_D \mathrm{MIN}\left[\frac{\pi}{4}, \frac{d}{k}\right] \tag{18}$$

Using $\pi k^2 n = \underline{P}$, this can be rewritten to incorporate the concept of fractional cover as:

$$f \approx \frac{8}{\pi}\underline{P} C_D \mathrm{MIN}\left[\frac{\pi}{4}, \frac{d}{k}\right] \tag{19}$$

For the range of flows of interest, the coefficient of drag for roughly spherical particles can be taken to be approximately equal to one. Note, however, that this represents a rather crude approximation to the more general case, in which C_D and A_F will both vary with the actual shape of the particles.

Equations 12, 15 and 19 provide estimates of the functional dependence of the frictional resistance on the inundation ratio for each of the flow regimes, as illustrated in Figure 3. The variation in this relationship with

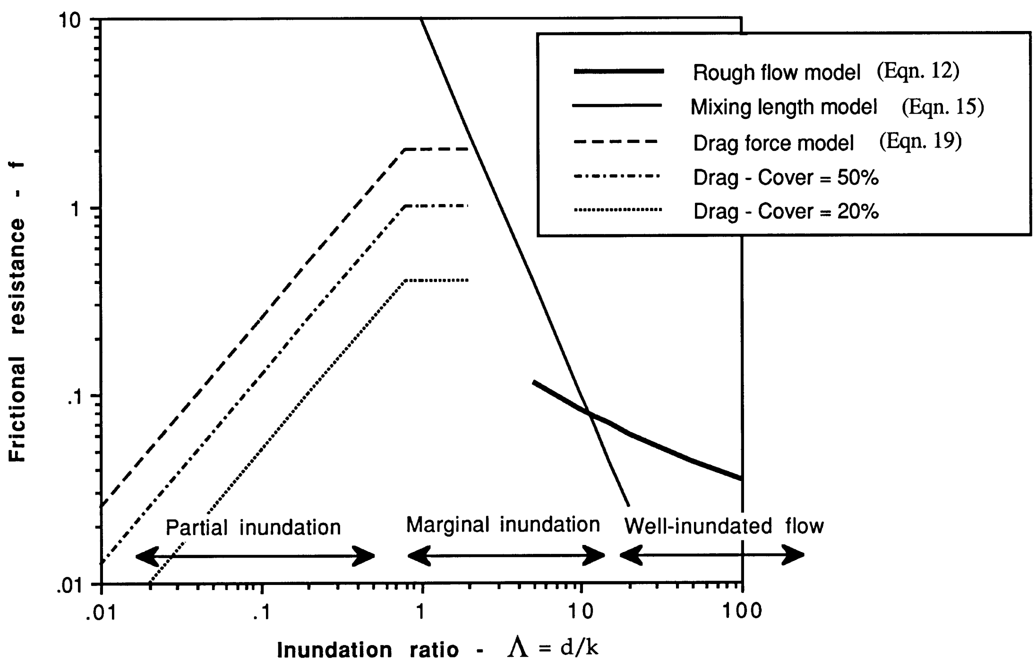

Figure 3. The approximate functional dependence of frictional resistance, f, on the inundation ratio $\Lambda = d/k$ for the three flow regimes

fractional cover $\underline{P}$ is also shown for the regime in which the elements are not yet fully submerged. As has been emphasized throughout the presentation of the modelling, these relationships are heuristic in character and have been developed to establish a basis for evaluating a wide range of data rather than to provide a rigorous analysis of a specific set of experiments. The objective of the proposed model, as given by these three equations, is thus to demonstrate the extent to which the inundation ratio represents a very useful variable which should not be neglected in favour of the Reynolds number in the evaluation of overland flow hydraulics.

COMPARISON OF MODEL WITH PUBLISHED DATA

Previously published studies reporting the hydraulic behaviour of overland flow over rough granular surfaces have been compiled to provide a comprehensive data set for the preliminary evaluation of the proposed model. In order to ensure some consistency between the relatively diverse data available in the literature and the assumptions underlying the model, the following criteria were applied in selecting appropriate field and laboratory data sets: (1) absence of rainfall, so that the overland flow was generated experimentally by a controlled and monitored upstream surface discharge; (2) dominance of grain roughness over vegetation on the surface, as the effects of vegetation are not explicitly accounted for in the present model; and (3) a reported measure of surface roughness from which an approximate inundation ratio could either be directly obtained or estimated. A summary of the data sets selected for the analysis is presented in Table I. In several studies (i.e. Savat, 1980; Rauws, 1988; Abrahams and Parsons, 1991; Gilley *et al*., 1992), the flow hydraulics are reported graphically rather than in tabular form, using plots of the frictional resistance as a function of Reynolds number.

Table I. Summary of field and laboratory data

Publication	Roughness scaling (cm)	Slope (tan θ)	Reynolds number	No. of data points
Abrahams *et al*. (1986)	Reported mean particle size: 0·683–4·133	0·1169–0·6873	843–4008	108 (Field)
Abrahams and Parsons (1991)	Reported mean gravel size: 0·837–1·25	0·232–0·084	~612–4018	73 (Field)
Abrahams *et al*. (1994)	Reported mean gravel size: 1·71–4·07	0·09–0·1636	860–4500 (reflects correction of reported values × 10)	54 (Field)
Bunte and Poesen (1993)	Reported mean diameter of intermediate axis: 1·45 Microscale–reported D_{50}: 0·009	0·022	920–3744	12 (Lab)
Bunte and Poesen (1993)	Reported D_{50} of intermediate axis: 1·5; 8·6 Microscale–reported D_{50}: 0·009	0·022	920–3055	24 (Lab)
Emmett (1970)	Reported mean particle size: 0·05	0·0179–0·0775	307–19615	45 (Lab)
Gilley *et al*. (1992)	Median diameter estimated from range of sizes reported: 3·18; 14·05	0·0135	~500–14889	100 (Lab)
Phelps (1975)	Reported mean grain diameter: 0·117	0·0083–0·0451	36–1600	46 (Lab)
Rauws (1988)	Reported diameter of uniform spheres: 1·6 Microscale–reported median diameter: 0·024; 0·118	0·017–0·174	~92–3634	204 (Lab)
Roels (1984)	Estimated approximate mean grain size based on data presented: 0·2	0·136; 0·178	72–14 144	97 (Field)
Savat (1980)	D_{50} estimated from particle size distribution, with two larger fractions weighted by 1·2 and 1·4 to account for aggregation: 0·003; 0·013; 0·038	0·008–0·580	~131–22 733	336 (Lab)

For those data sets, numerical values for the Reynolds number and the frictional resistance were extracted by digitizing the graphical data. In other cases, these values were reported numerically in tabular form and so could be used directly in the analysis. For those studies in which it was not reported explicitly, the depth of flow was back-calculated based on the Reynolds number and the frictional resistance for each data point, according to:

$$d = \frac{\nu Re}{4V}$$

where (20)

$$V = \left[\frac{2g\nu \sin\theta Re}{f} \right]^{1/3}$$

and the kinematic viscosity ν was taken to be constant and approximated as $0{\cdot}01\,\mathrm{cm}^2\,\mathrm{s}^{-1}$. As the kinematic viscosity varies slightly with temperature, this approximation can be expected to introduce a small amount of error into the calculated values of flow depth.

The roughness scalings described in Table I, together with the depth of flow, were used to estimate the inundation ratio for each data point. Although the actual scalings are somewhat variable between data sets, the selected measures all describe either the mean or median diameter of the surface grain roughness, rather than reflecting extreme values, and thus should provide a general indication of the ‘average’ surface inundation for each data point. In several of the laboratory experimental studies (e.g. Bunte and Poesen, 1993, 1994; Gilley *et al*., 1992; Rauws, 1988), the surface roughness is characterized by two discrete length-scales, a fine-grained microscale roughness onto which coarser-grained particles are embedded or emplaced, rather than a single mean or median grain size. For those cases, the D_{50} associated with the large scale or ‘macroscale’ roughness was used in calculating the inundation ratio for each flow. However, several of those studies systematically varied the percentage of the surface occupied by the larger size fraction, so that it is anticipated that these data will exhibit a significant degree of scatter when scaled by a constant measure of surface roughness.

The frictional resistance, *f*, is illustrated in Figure 4 as a function of estimated inundation ratio, Λ, for all of the compiled data. Overall, the correspondence between the proposed models and the field and laboratory data is quite good, particularly considering the diversity of the field settings and experimental techniques underlying the observational data. Although there is admittedly a large degree of scatter in the data, the generally non-monotonic relationship between the frictional resistance and the scaled flow depth is apparent. The amount of scatter is reduced significantly when one considers the data which are characterized by a single, rather than a distinctly bimodal, grain roughness scale as illustrated in Figure 5. Where vegetation is present on the surface (e.g. Abrahams *et al*., 1986, 1994), the frictional resistance should generally be greater than predicted, as its added effect on flow resistance is not accounted for by the theory. This tendency is also apparent in Figures 4 and 5.

In conjunction with the generally good correspondence between the model predictions and the compiled observational data, there are also several systematic discrepancies. At values of the inundation ratio less than one, the data tend to be much more dispersed than they are for the inundated flows. This is not surprising in that, in addition to the surface variability introduced by the presence of vegetation and changes in the percentage of the surface covered by macroscale roughness, the reported flow hydraulics may be significantly modified by other details, such as the shape and orientation of surface particles (upon which the coefficient of drag and projected frontal area are dependent) and also by the manner in which the mean depth of flow is actually estimated (Abrahams and Parsons, 1990). For the partially inundated flows, the effects of a variable fractional cover can be evaluated using Equation 19 with appropriate values for $\underline{P}$. The results which are obtained using this approach are compiled in Figure 6 for those sets of laboratory data where this parameter is systematically varied (i.e. Bunte and Poesen, 1993, 1994; Gilley *et al*., 1992). The trend illustrated is as anticipated in that increasing fractional cover is associated with a greater resistance to flow. However, for most of the data the frictional resistance is still generally underpredicted by at least a factor of two. This indicates that the effects of

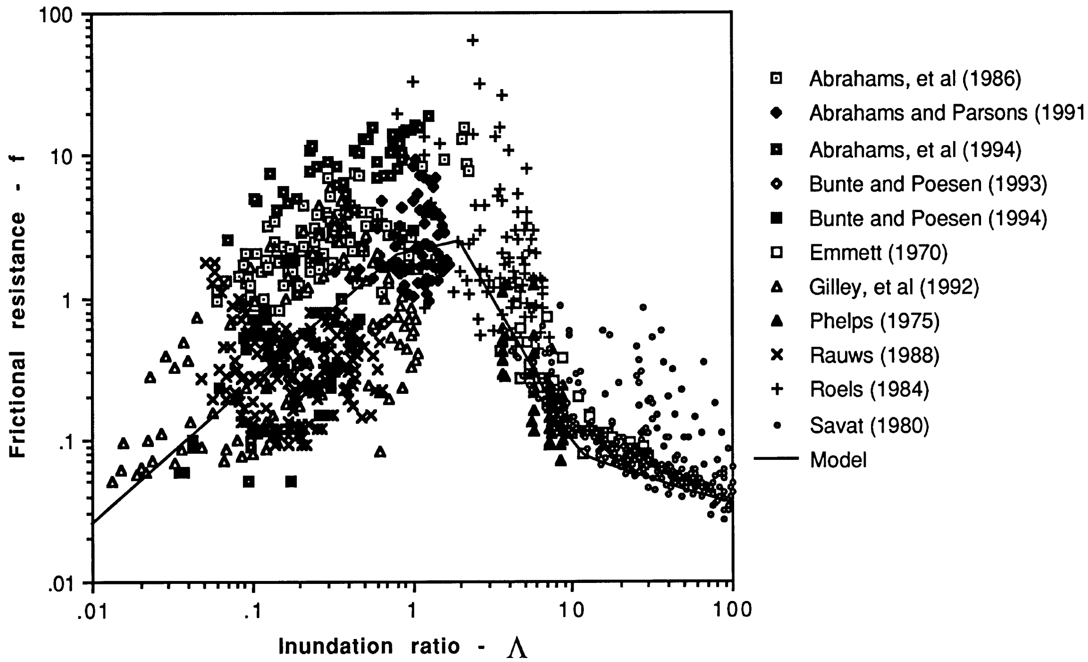

Figure 4. Frictional resistance as a function of flow inundation for the data sets indicated. The methods used for calculating the inundation ratio are described in the text and in Table I

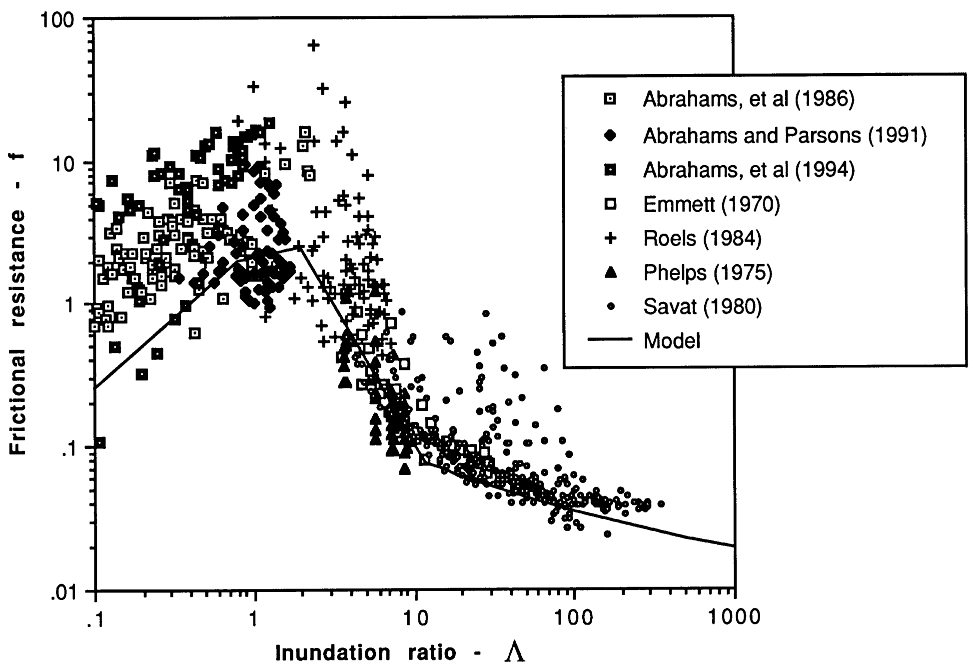

Figure 5. Frictional resistance as a function of flow inundation for the unimodal data only

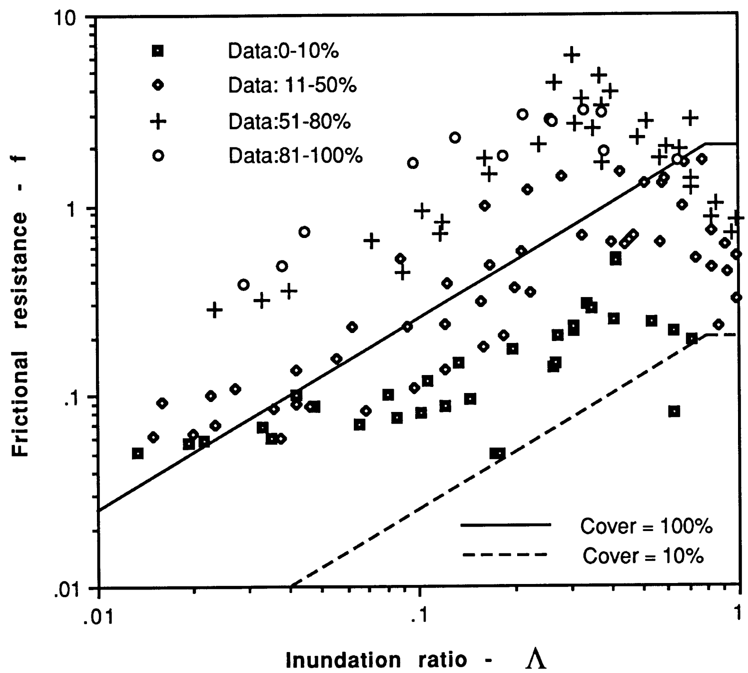

Figure 6. Frictional resistance as a function of flow inundation for data sets with a reported variable percentage cover. The data presented were compiled from Gilley *et al.* (1992) and Bunte and Poesen (1993, 1994)

the roughness elements on the overall structure of the flow are not fully accounted for by a model that is based on flow around individual elements and that uses a coefficient of drag originally developed for fully submerged objects in an otherwise uniform flow field. The presence of a free surface in this regime will presumably increase the coefficient of drag as the flow will be characterized by local variations in the height of the water surface as flow is diverted around roughness elements. In some cases, these flow depth variations may be associated with small hydraulic jumps in the lee of each element, which would contribute significantly to energy dissipation and, accordingly, frictional resistance in the flow. Additionally, some of the data compiled from Gilley *et al.* (1992) exhibit a peak in frictional resistance at an inundation ratio significantly less than one, suggesting that the roughness scaling used for those flows results in an underestimation of the average surface inundation. This tendency was also observed in experimental studies by Abrahams and Hirsch, and in that case seemed to reflect a local superelevation of the fluid surface immediately upstream of each roughness element impeding the flow (A. D. Abrahams, personal communication, 1995).

At higher values of the inundation ratio, two systematic discrepancies between the proposed model and the experimental data are apparent. Firstly, the mixing length theory proposed for the marginally inundated flows (i.e. d/k from 1–10) seems to slightly underestimate the rapid rate of increase in frictional resistance as the flow becomes increasingly shallow and the inundation ratio approaches one. This is possible due to contributions of form resistance and wave drag which are not explicitly accounted for in this regime. It may additionally reflect the development of a multilayer separated flow during marginal inundation that is characterized by a zone of very rapid flow above the elements, which is partially lubricated by relatively stagnant fluid trapped in a lower layer between the elements. However, the degree of the disparity between the predicted and observed trend is not sufficient to completely dismiss the modified mixing length theory used to model the flow. There is also, in general, a relative paucity of reliable data available for evaluating this regime, particularly in the transitional region $d \cong k$. The second discrepancy is suggested by the several data points at high inundation ratios which lie

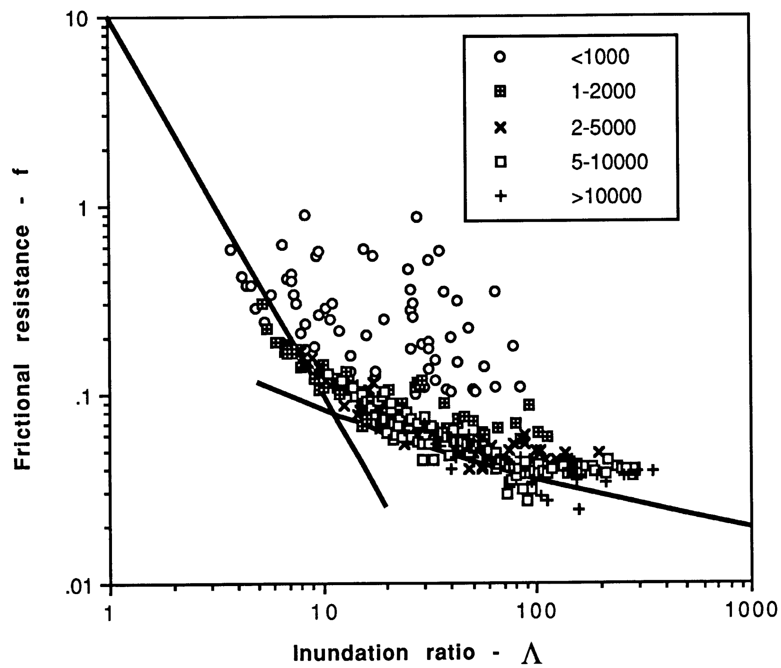

Figure 7. Frictional resistance as a function of flow inundation for well-inundated flows with variable ranges of Reynolds number, as indicated

significantly above the predicted value for that range of flows. When one examines the individual data sets exhibiting this behaviour, as illustrated in Figure 7, it appears that the tendency for this trend is related to the Reynolds number of the flow. In particular, well-inundated flows with Reynolds numbers below approximately 1000, are not fully described by the proposed model and tend to exhibit values of frictional resistance above that which would be predicted solely on the basis of the inundation ratio. This lack of correspondence is anticipated as these flows do not represent a rough turbulent flow regime.

DISCUSSION AND CONCLUSIONS

The physical modelling presented above strongly indicates that a fundamental dimensionless parameter for evaluating overland flow hydraulics is a measure of the extent of the inundation of the surface roughness, as this parameter determines the dominant physical mechanism controlling the frictional resistance to flow. Although the data presented in Figure 4 exhibit a significant degree of scatter relative to the heuristic theory proposed, they nevertheless illustrate the utility of this approach and suggest that further efforts should be directed towards the more complete development of the theory. In contrast to this approach, Figure 8 illustrates the frictional resistance as a function of the Reynolds number for the same data points used in the analysis of the inundation model and visually demonstrates the limited values of the more traditional method, particularly if one is interested in comparing flow hydraulics associated with diverse field environments. Although several individual data sets illustrated in Figure 8 suggest a generally decreasing frictional resistance with Reynolds number, when all the data are considered as a whole, this trend is no longer apparent.

The use of an inundation ratio for describing overland flow hydraulics is also supported by the results of multivariate regression analysis of the data set, as reported in Table II. For the analyses, each of the three flow regimes (i.e. $\Lambda < 1$, $\Lambda = 1$–10, and $\Lambda > 10$) was evaluated using separate regressions that considered the relative contributions of the inundation ratio, the Reynolds number, the slope and the percentage cover, in explaining

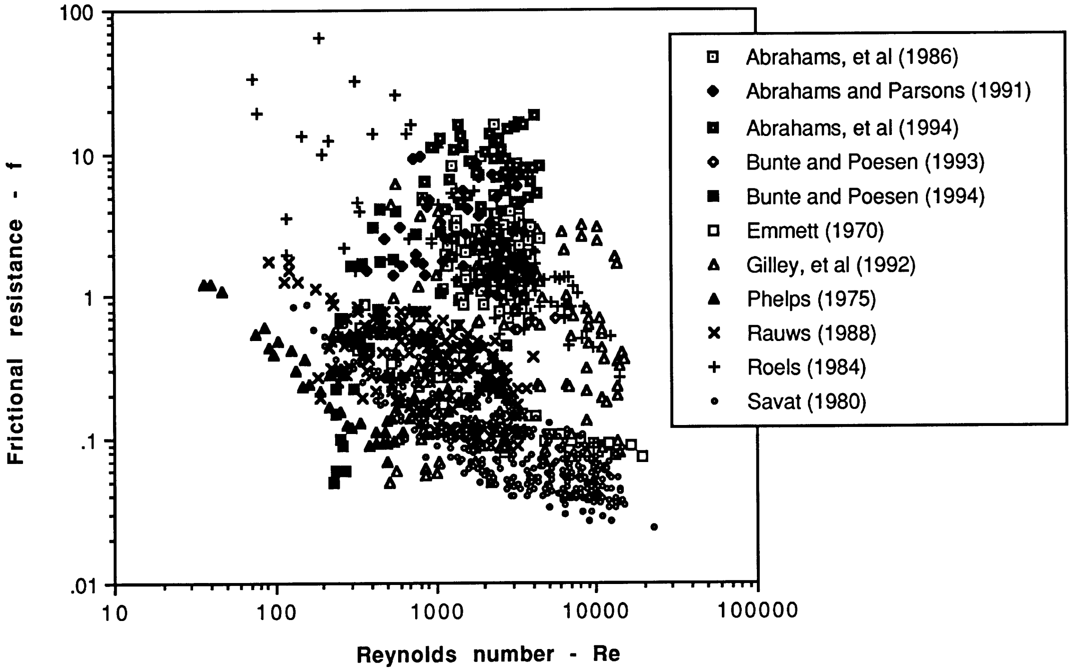

Figure 8. Frictional resistance as a function of Reynolds number for all compiled data (cf. Figure 4)

Table II. Results of stepwise multivariate regression analyses

Data set	Dependent variable	Independent variables (in order of entering regression)	R^2–total; standard error	No. of data points
1. Inundation ratio <1 Unimodal data only	$\log f$	[$\log\Lambda$, $\log S$, $\log Re$]	0·713; 0·348	227
2. Inundation ratio <1 Bimodal data with variable cover	$\log f$	[$\underline{P}$, $\log\Lambda$, $\log Re$, $\log S$]	0·824; 0·234	121
3. Inundation ratio <1 All data	$\log f$	[$\underline{P}$, $\log\Lambda$, $\log S$, $\log Re$]	0·652; 0·361	564
4. Inundation ratio = 1–10	$\log f$	[$\log\Lambda$, $\log S$]	0·702; 0·340	240
5. Inundation ratio >10 All data	$f^{-1/2}$	[$\log Re$, $\log\Lambda$]	0·817; 0·428	294
6. Inundation ratio >10 Reynolds number >2000	$f^{-1/2}$	[$\log\Lambda$, $\log Re$]	0·774; 0·125	249

variations in the frictional resistance to flow. The independent variables which contribute to explaining the variance in the friction factor, f, such that their inclusion is significant at the 0·01 level, are indicated in Table II for each regine, as is the total R^2 value and the standard error of estimate for each analysis.

For all cases considered, the inundation ratio contributed significantly to explaining the variation in frictional resistance, and was either the first or second variable to enter the regression. The total R^2 values for the multiple regressions for each regime are 0·652, 0·702 and 0·817 for theΛ <1, Λ = 1–10 and the Λ >10 cases, respectively. For the partially inundated regime, i.e. where $\Lambda < 1$, the percentage cover was the first variable to enter the regression. The data for this regime was therefore subdivided to separately evaluate the 'unimodal' data, in which the surface grain roughness is described by a single parameter, and the 'bimodal' data, in which two distinct length-scales are present and the precentage cover of the larger size fraction is systematically varied

between experimental runs. (A third group consisting entirely of Rauws' (1988) data was not included in the further analysis. Although those data can be classified as bimodal, the percentage cover of the larger fraction is quite small and was held constant during the experiments.) This breakdown of the data set increased the R^2 values to 0·713 and 0·824 for the unimodal and bimodal data, respectively. For the unimodal case, the inundation ratio became the first variable to enter the regression, and the surface slope was the second, contributing 31 per cent and a further 37 per cent, respectively. The Reynolds number was the third variable and only contributed a 0·3 per cent improvement in the explanation of the variance. The importance of the surface slope in explaining the variance is possibly an indication of the contribution of wave drag to frictional resistance in these very shallow flows. However, for the case of the bimodal data, the surface slope was the fourth variable to enter the regression, following the percentage cover, the inundation ratio and the Reynolds number. This may, though, reflect the limited variation in slope underlying the data in that each data set was based on experiments performed at a constant value of slope, with varying percentage cover. Furthermore, the slopes represented by the bimodal data are very low, ranging from 0·0135 to 0·022, and thus should not be expected to be associated with high values of wave resistance.

The marginally inundated regime, for which Λ = 1–10, exhibits relatively strong correlations between frictional resistance and the inundation ratio and the surface slope, with the dependent variables contributing 50 per cent and 20 per cent, respectively, to the explanation of the total variance in frictional resistance. As anticipated, the coefficient for the inundation ratio indicated a decrease in frictional resistance with increasing inundation. For this regime, the Reynolds number did not contribute to explaining the variance at a level which was of statistical significance. For the well-inundated flows, however, the Reynolds number was the first variable to enter the regression when all of the data for $\Lambda > 10$ were considered. This clearly reflects the trends illustrated in Figure 7, in which flows at lower Reynolds numbers exhibited much higher frictional resistance. A second regression, which considered only the data for which the Reynolds number was greater than 2000, was also performed and in this case the inundation ratio was the primary explanatory variable, and the Reynolds number made a only small contribution to the total R^2 of 0·774. The surface slope did not contribute at a level which was of significance in either of the cases considered, despite the fact that the slope is systematically varied in the experiments from which these data were derived. This is also not surprising, however, as the potential contribution of wave drag associated with large-scale roughness, if present, may only be apparent in relatively shallow, rather than well-inundated, flows.

The results of both the theoretical and the statistical analyses underscore the relevance of the inundation ratio in evaluating overland flow hydraulics and the need for further work in this area. In addition to developing the physical model to incorporate the increased resistance associated with the presence of vegetation and variations in resistance introduced by the shape of roughness elements, there are other effects not discussed thus far, such as rainfall impact (e.g. Savat, 1977; Guy *et al*., 1990) and spatially varying infiltration (Dunne *et al*., 1991; Motha and Wigham, 1995), which clearly contribute to variations in resistance to flow over natural surfaces and should be considered in further modelling efforts. The need for additional experimental work is also strongly indicated by the results. In particular, flows at or near marginal inundation are not represented by sufficient data, their hydraulics are not well understood. Similarly, the behaviour of partially inundated flows and the contributions of various sources of surface roughness to frictional resistance in these flows, especially surface slope and associated wave drag, warrant more detailed experimental investigation. Although an abundance of data exist for this regime, the physical factors contributing to frictional resistance are not simply additive (Abrahams and Parsons, 1994; Gilley and Kottwitz, 1995) such that flow resistance can be very difficult to predict *a priori*. The methodology proposed herein, i.e. that of distinguishing flow regimes and evaluating frictional resistance based on the average degree of inundation of the surface rather than the flow Reynolds number, should be utilized in future experimental work on this topic as it provides a physical basis for evaluating and comparing flow hydraulics over a wide range of field conditions.

NOTATION

A_F = Projected frontal area of roughness element (L^2)
B = Constant factor in turbulent velocity profile near boundary (dimensionless)

C_D = Coefficient of drag on roughness element (dimensionless)
D = Representative length-scale (diameter) of roughness element (L)
D_{50} = Diameter of 50th percentile class of particle size distribution (L)
D_{85} = Diameter of 85th percentile class of particle size distribution (L)
d = Depth of flow (L)
e = Roughness length-scale for pipe flow and open channel flow (L)
F_D = Drag force introduced by roughness element (MLT^{-2})
Fr = Froude number (dimensionless)
f = Friction factor (dimensionless)
g = Gravitational acceleration (LT^{-2})
k = Characteristic roughness length-scale for overland flow (L)
l = Mixing length-scale in vicinity of roughness elements (L)
n = Number of roughness elements per unit area (L^{-2})
P = Percentage of surface covered by roughness elements (dimensionless)
$\bar{q}$ = Specific discharge (L^2T^{-1})
Re = Flow Reynolds number (dimensionless
r = Radius of roughness element (L)
u = Downstream component of velocity (LT^{-1})
V = Average flow velocity (LT^{-1})
V^* = Flow shear velocity near boundary (LT^{-1})
y = Vertical coordinate for velocity profile (L)
y^* = Vertical coordinate scaled by roughness length-scale near boundary (L)
ε = Roughness ratio for pipe flow (dimensionless)
θ = Angle of surface slope (dimensionless)
κ = von Karmen's constant in turbulent velocity profile (dimensionless)
Λ = Inundation ratio for overland flow (dimensionless)
μ = Fluid dynamic viscosity ($ML^{-1}T^{-1}$)
ν = Fluid kinematic viscosity (L^2T^{-1})
ν_T = Turbulent kinematic viscosity (L^2T^{-1})
ρ = Fluid density (ML^{-3})
τ = Shear stress in fluid ($ML^{-1}T^{-2}$)
τ_0 = Shear stress exerted at the boundary ($ML^{-1}T^{-2}$)

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